Cones, Disks, Wheels and Spheres for Area and Integration from Bavaria to Boston and beyond

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Errata

- Page 5: To the figure of Hermann's planimeter add: "© Deutsches Museum, München"
- Page 8: Change "the wheel's rotation is proportional to both the x- and to the y- movement" to: "the wheel's rotation is proportional to both the x-change dx and to the y-movement"
- Page 28: Change "$f(x) = \frac{\alpha_0}{2} + \Sigma_{k\geq 1}(\alpha_k \sin(kx) + \beta_k \cos(kx))$" to: "$f(x) = \frac{\alpha_0}{2} + \Sigma_{k\geq 1}(\alpha_k \cos(kx) + \beta_k \sin(kx))$"

Contents

Abstract

The very first planimeter, invented in 1814 by Johann Martin Hermann in Bavaria, used a cone wheel gear as an integrating mechanism for area measurements. The same principle was implemented in high precision instruments with up to 18 integrators initiated by Vannevar Bush for evaluating differential equations in higher physics.

This article presents the history of these instruments and their inventors. It illustrates the rise of a simple kinematic principle from measuring devices to mechanical supercomputers.

1 The First Generation: Cones, Disks and Wheels

It was in the first half of the 19th century when efforts for tax equality and the rise of cadastral authorities in Europe led to an urgent need for devices that could simplify the calculation of areas of real estate.

The first apparatuses were simple analog devices which assisted the calculating clerk in taking the measurements and sometimes multiplying two quantities. These instruments were either only approximating the area or they were restricted in the type of region they could handle.

The simplest devices are so called harp planimeters. Harp planimeters divide the area to be measured into narrow stripes of equal width. The sum of the lengths of the stripes multiplied by their width gives an approximation for the demanded area.

For more information on these non-integrating planime-

ters see [DH09].
Most of these instruments were cumbersome to operate and none of them could handle arbitrarily shaped areas.

1.1 The First Integrating Mechanism

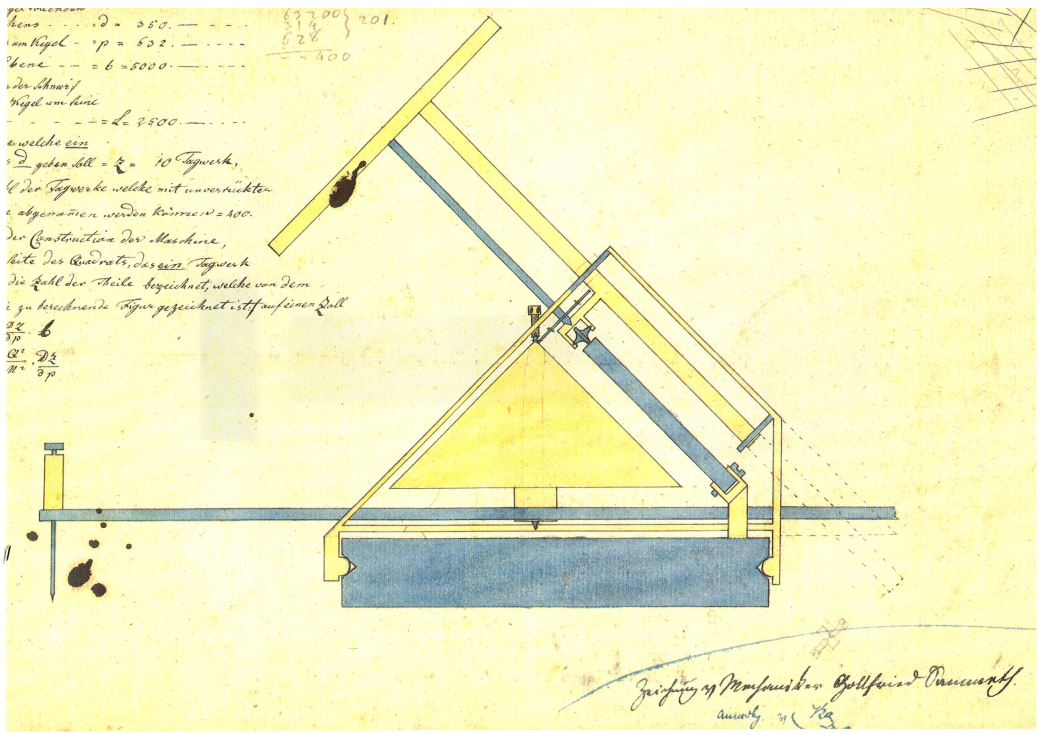


Figure 1: Original diagram of Hermann's planimeter. Courtesy of Professor Fischer, München

The first to invent a functioning integrating device which could calculate the area within any arbitrarily shaped closed curve by tracing a pointer around it was the German land surveyor Johann Martin Hermann.

Hermann's construction dates back to 1814, but since he abstained from publishing it, his invention did not receive closer attention at that time. The prototype of Hermann's planimeter was disposed as scrap metal, and the inventor's manuscripts were forgotten for about 40 years. [Fis95, Bau55].

Fortunately a drawing of the planimeter has survived which clearly shows the underlying principle (Figure 1).

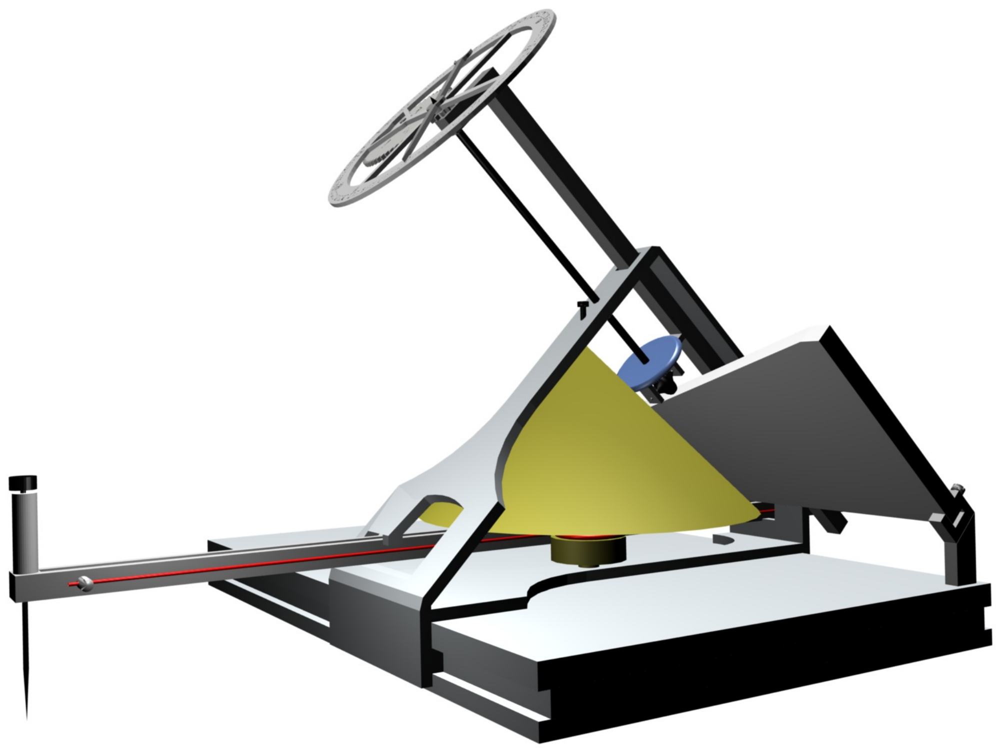


Figure 2: 3D model of Hermann's planimeter

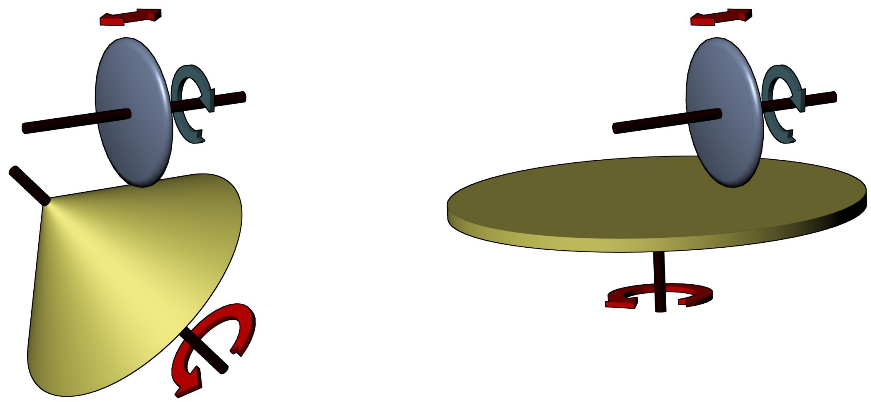
The central element in Hermann's planimeter is a cone which is mounted in an upright position in a frame. A pointer that moves freely on the map in x-direction causes the cone to rotate, proportionally with the x-value of the pointer. A recording wheel at the surface of the cone is driven by friction and thus is caused by the rotation of the cone to rotate likewise. Cone, pointer and recording wheel are mounted in a carriage which can be moved in y-direction. The recording wheel is guided via a ramp in a manner that it is at the top of the cone when $y = 0$ and moves with increasing y towards the base of the cone (Figure 2).

To make clear that this is enough to measure arbitrary areas, theoretically exact, with an accuracy and precision that is only limited by the accuracy of construction, we shall take a closer look at the kinematic heart of Hermann's instrument.

1.2 Cone Wheel Planimeters

A simple method to measure the length of a curved line is to roll a wheel along the line and to add up the number of rotations. This was already described by the Roman engineer Vitruvius in chapter nine of the tenth book of his “De Architectura libri decem”.

If this recording wheel is combined with a spinning cone, its speed depends on its position at the cone. It



(a) Cone wheel mechanism (b) Disk wheel mechanism

Figure 3: From cone to disk

will rotate fast if it is located near the base of the cone, and its speed will decrease if it is moved towards the tip (Figure 3). Therefore, if the cone is rolled along the x-axis and the wheel, without loosing its contact to the cone, is moved parallel to the y-axis, the wheel's rotation is proportional to both the x- and to the y-movement, thus continuously adding up the product of both, resulting in $\int y \, \mathrm{d}x$.

Hermann's planimeter vanished from the scene before it received any attention, but the need for area measuring devices was increasing. Almost simultaneously, the very same mechanism got re-invented at several places all over Western Europe.

In 1824, 10 years after Hermann, the mathematician Tito Gonnella from Tuscany, who certainly could not have had any knowledge of Hermann's device, constructed an integrating device that was based upon the cone wheel principle [Fis02]. One year later he presented an article "Teoria e descrizione d'una macchina colla quale si quadrano le superficie piane" [Gon25], which can be regarded as the first publication on integrating planimeters.

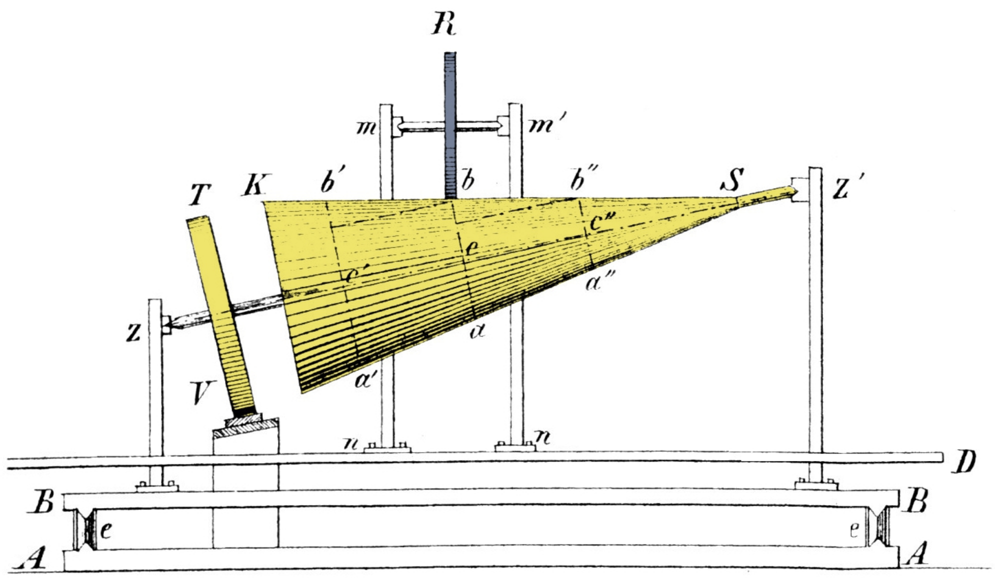


Figure 4: Ernst's planimeter [Bau53, modified]

In 1827, the Swiss engineer Johannes Oppikofer also used the combination of cone and friction wheel for his planimeter. This instrument was later improved by the German mechanic Heinrich Rudolf Ernst in Paris [Bau53] (Figure 4).

And in 1851 the mechanism experienced its fourth independent re-invention by Scotsman John Sang from Kirkaldy, who named his instrument *Platometer* [San51] (Figure 7).

1.3 Disk Wheel Planimeters

Along with the cone wheel planimeters, another variation of integrating mechanisms using a disk instead of a cone was developed.

One can easily see that the integrating principle is independent from the opening angle of the cone. That angle can even be extended up to $180°$, thus degenerating the cone to a disk (Figure 3).

The first to realize that a disk is but a special type of cone, was the above-mentioned Italian Professor Tito Gonnella from Florence, who had already suggested a disk wheel mechanism in his 1825 paper [Gon25, pp. 125–126].

In approximately 1850, the Swiss engineer Caspar Wetli also constructed a disk wheel device that was later improved by the German astronomer Peter Andreas Hansen from Gotha.

These planimeters all had something in common; that an area, which is an integral, $A = \int y \, \mathrm{d}x$, was calculated by separating the x- and the y-part of the coordinates

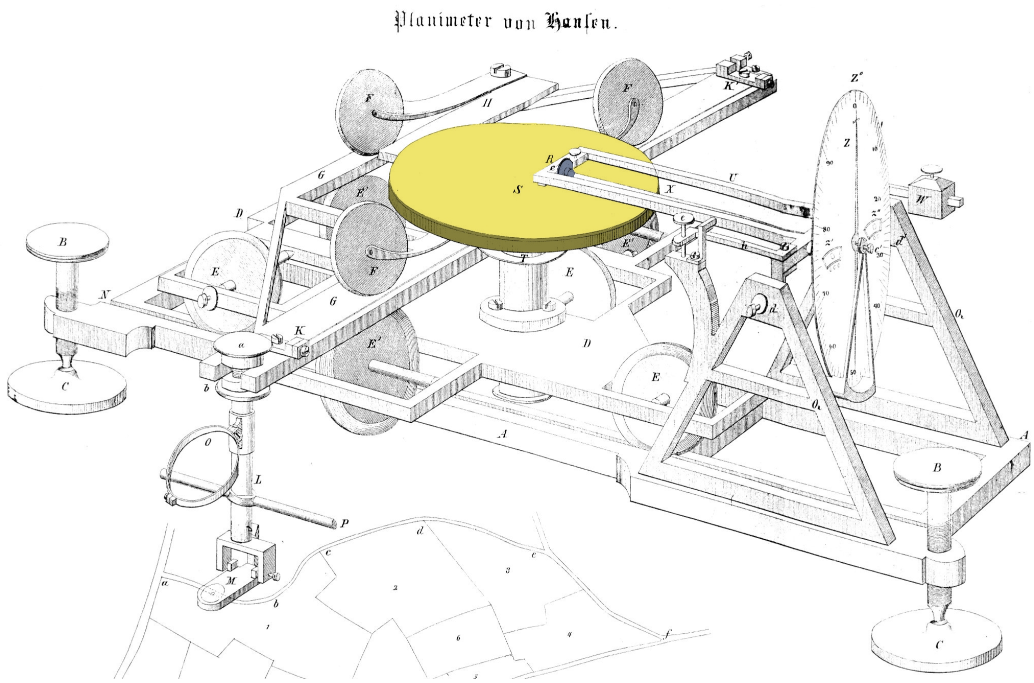


Figure 5: Hansen's planimeter [Bau53, modified]

of a tracing pointer, rotating with the x-part a cone or a disk and determining with the y-part a transmission ratio proportional to the y-value. Because of this principle these planimeters are called *Coordinate Planimeters*.

Later, there were also attempts to use polar coordinates instead of the cartesian ones and to calculate the area by the formula $A = \frac{1}{2} \int r^2 \, \mathrm{d}\varphi$, but those devices did not rise to prominence [Dre10].

1.4 Dynamometers

The need for the exact determination of areas did not only emerge in land surveying. The industrial revolution had changed the status of engineering from craftsmanship to science, and the rise of powerful engines created a demand for devices that could objectively calculate their degree of efficiency, not least for economical reasons.

To measure the work of a machine one has to to integrate the force over distance or to integrate the power over time. Such measuring devices were called *Dynamometers*.

The first dynamometers used the same kinematic principle for integrating as the early planimeters did.

In 1837, the French physicist Arthur Morin described a device for "mesurer la force des moteurs animés ou les efforts de traction ainsi que les quantités de travail q'ils développent" that made use of a disk wheel integrator, which he called a *Compteur* [Mor37]. Morin mentions that he got the idea for his device by "mon maître et mon ami" the French mathematician Jean-Victor Poncelet [Mor41].

Indeed Poncelet presented a cone wheel mechanism and a disk wheel mechanism in his "Mécanique industrielle", a book developed from lectures he had held from 1827 to 1830 in Metz. However Poncelet had at that time only used these mechanisms as variable transmission gears and not yet for integrating [Pon39].

Later, in 1841, Morin was already familiar with the cone wheel planimeter invented by Ernst and suggested using that mechanism in dynamometers [Mor41].

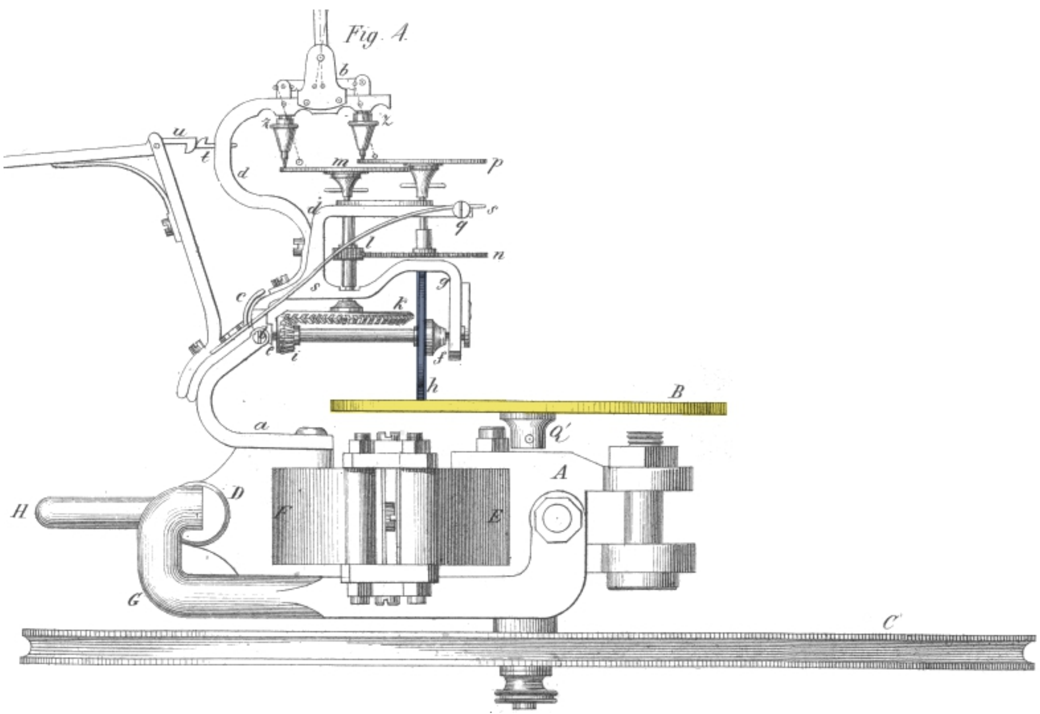


Figure 6: Morin's mechanism [Mor37, modified]

In 1842, the Reverend Henry Moseley, Professor of Natural Philosophy and Astronomy at King's College in London, who knew Morin's work, presented a device for "Calculating the Numerical Values of Definite Integrals" to the British Association for the Advancement of Science. This disk wheel mechanism combined with a counter, as Moseley claimed, "had nothing in common with the Compteur of M. Morin except the fruitful and admirable principle of M. Poncelet" [Mos42a].

In the same year a Committee appointed at the Tenth Meeting of the British Association for the Advancement of Science evaluated a "Constant Indicator for Steam Engines", which had also been invented by Professor Moseley [Mos42b]. This dynamometer used a cone instead of a disk for integrating.

This means that the same kinematic element had been invented all over Western Europe for different purposes, while the various inventors knew nothing or little of each other. This came to a sudden change in the year 1851.

2 The Great Exhibition

It was in the year 1851 when the Great Exhibition took place in London. This exhibition, the first in a series of international trade fairs, was a superlative in every possible aspect. To host the exposition, the Crystal Palace, an enormous glass house, was erected, "covering a space more than 18 acres, measuring 1,851 feet in length, and 456 in extreme breadth, capable of containing 40,000 visitors and affording the frontage for the Exhibition of Goods to the extents of more than 10 miles"[Com51].

A contemporary predicted, "...we may safely pronounce, that the House of Glass will exist in the annals of history long after the vaunted pyramids of Egypt of which the builders and the object are already alike unknown, shall have crumbled into dust." [Tal51, vol. 1, p. iii]. This turned out to be somewhat hopeful as the Crystal Palace was destroyed by a fire in 1936.

As for the exhibits, it was reported: "One of the distinguishing characteristics of the Great Exhibition was its vast comprehensiveness. Nothing was too stupendous, too rare, or too costly for its acquisition; nothing too minute or apparently too insignificant for its consideration. Every possible invention and appliance for the service of man found a place within its all embracing limits; every realization of human genius, every effort of human industry might be contemplated therein, from the most consummate elaboration of the profoundest intellect, to the simplest contrivance of uneducated thought ... from the wondrous calculating machine, down to the simplest toy" [Tal51, vol. 1, p. 207].

More than 100,000 exhibits were presented by about

15,000 exhibitors. About half of them were British [Com51].

The exhibits were divided into four categories: raw materials, machinery, manufactures, and fine arts. The categories were subdivided into 30 classes. In each class awards were granted for the most outstanding exhibits.

2.1 Planimeters at the Great Exhibition

Planimeters, together with watches and pianos, had their place in class X (Philosophical, Musical, Horological and Surgical Instruments), which comprised state of the art science and technology. In this class, instruments by Sang, Gonnella, Wetli and Hansen together with two non-integrating area calculating devices from France were presented. One of the latter was the mechanised harp planimeter invented by the Chef de Cadastre Beuvière, the other one a hyperbolic grid by Jean Antoine Laur, a surveyor and professor of geodesy.

The planimeters by Hansen (produced and presented by instrument maker Ausfeld), Sang and Laur were awarded Honourable Mention by the jury. Caspar Wetli was given a Price Medal, and Gonnella received the highest award, a Council Medal [Com52, p. 304].

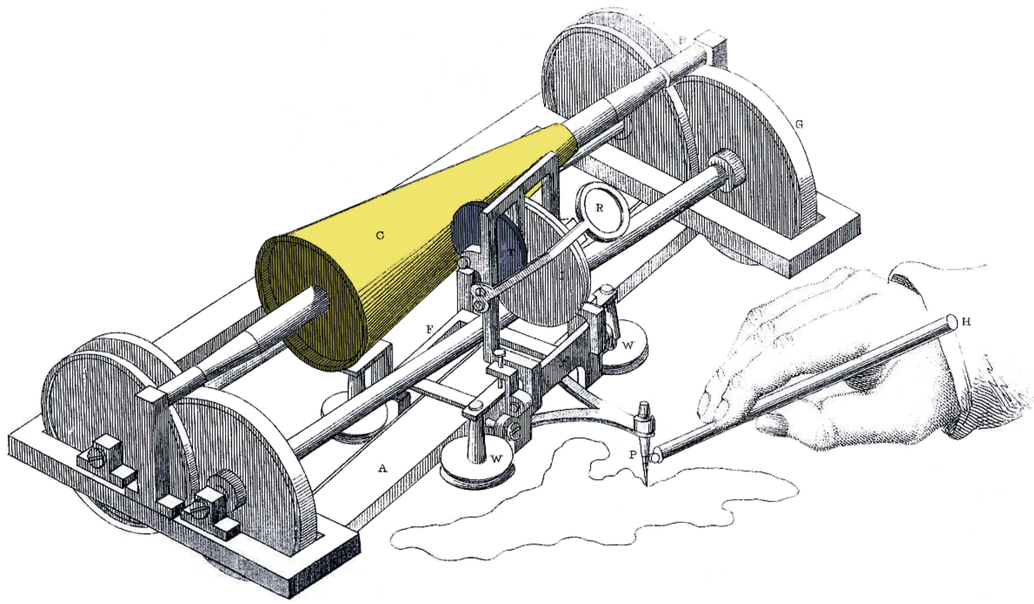


Figure 7: Sang's platometer [San52, modified]

2.2 Dynamometers at the Great Exhibition

Dynamometric instruments which contained integrating mechanisms were also represented at the exhibition. Dynamometers were not regarded as philosophical instruments, but classed in class V (Machines for Direct Use, including Carriages and Railway and Naval Mechanism), section G (Weighing, Measuring and Registering Machines for Commercial and not for Philosophical Purposes).

Here the instrument maker P. Claire from Paris showed, among other devices, a rotary dynamometer designed by Poncelet and Morin [Com52, p. 191].

Moreover, the jury for that class V (Morin being Vice-Chairman) was assisted by Morin's dynamometer in measuring the quality of various machines [Com52, p. 181].

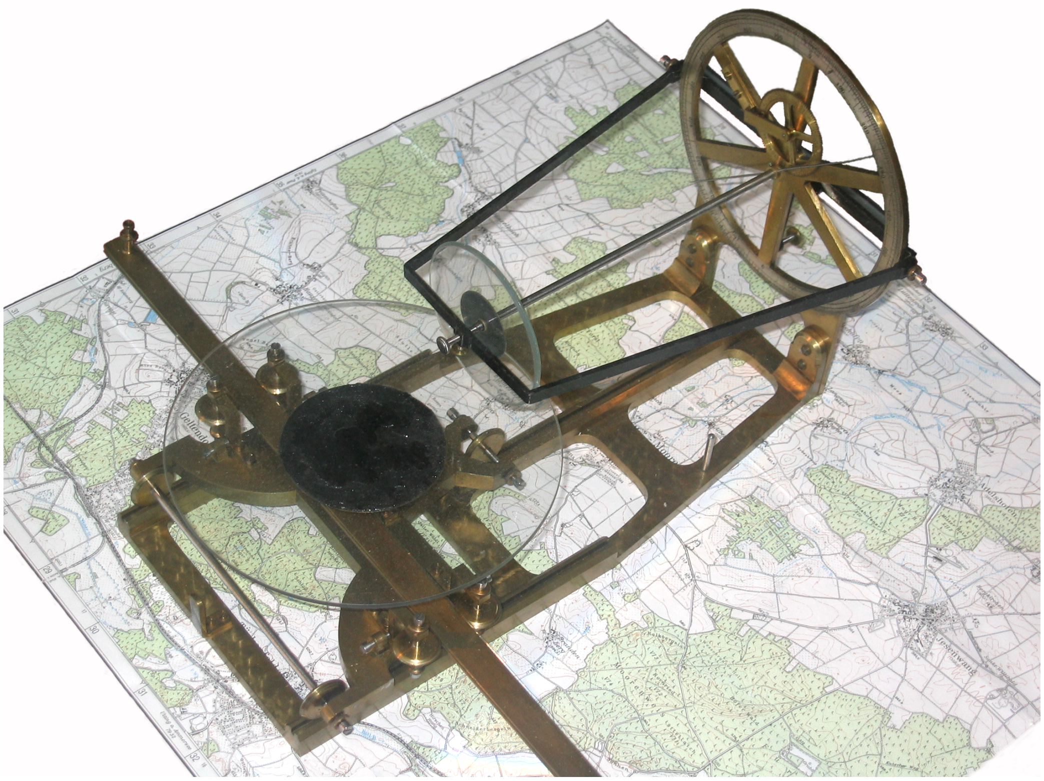


Figure 8: Wetli's planimeter. Deutsches Museum, München

2.3 Writing the History of Planimeters

Whereas before the Great Exhibition there were several isolated lines of integration mechanisms development in individual countries and application areas, the exhibition itself served as an information hub, bringing all these lines into together. It also raised the question as to who was the actual inventor of integrating mechanisms.

In 1852 a committee of the Royal Scottish Society described Sang's platometer and, despite "None of the members of your Committee having seen the foreign instruments, and not knowing any one who had" they came to the conclusion, that "even should it turn out that the French instrument was similar in principle (of which Your Committee have no evidence), yet they have a thorough conviction that, as far as Mr Sang is concerned, the merit of originality would no less belong to him" [Gra52].

In Germany in 1853, the geodesist Carl Maximilan von Bauernfeind published an article; "Die Planimeter von Ernst, Wetli und Hansen, welche den Flächeninhalt ebener Figuren durch das Umfahren des Umfangs angeben" in which he described the devices known to him (Hermann's planimeter was at that time still unknown to the world) and offered a list of several possible applications beyond the cadastral field. He deliberately omitted Sang's planimeter, since it had nothing new to offer [Bau53]. Two years later Bauernfeind added another piece of knowledge to the history of planimeters. He could now give an elaborately description of Hermann's planimeter and ascertained Hermann's priority [Bau55].

In his book pulished in 1857, "Rapport sur les machines et outils employés dans les manufactures", a report on the Great Exhibition, Poncelet credited the French engineer Jean-Baptiste Laborde with the invention of the friction wheel. In 1824, Laborde had a spinning machine patented, which used both, a cone and a disk wheel mechanism, to keep the speed of the spools at a constant level [Pon57]. Again, this was a variable transmission gear, and as such, an important step towards integrating mechanisms, but it was not yet used for integration.

Later, in 1873, the Mathematician Antonio Favaro from Padua published a history of planimeters in which he gave the credit to Gonnella for being the planimeter's inventor. In his article, after a lengthy description of Gonnella's device, he came to the conclusion that Hermann's invention was only of historical interest "... dass der letzteren nur ein historisches Interesse beigemessen werden kann, während das Verdienst um die wirkliche Urheberschaft erwähnter Instrumente Jenem gebührt, der sie nicht bloß erfunden, sondern auch dem Gebrauche durch Veröffentlichung zuerst zugänglich gemacht hat" (that his invention can only be attributed to historical interest, as the real inventor must be credited not only with the invention itself, but also with the application and accessibility through publication of those devices) [Fav73]. This inventor is, as Favaro emphasizes, Gonnella.

While the discussion about the roots of the coordinate planimeter was still under way, the chapter of its history could already be closed. In 1854, Jacob Amsler from Switzerland had invented the *Polar Planimeter*, a device which was much easier to construct, hence cheaper, and which in cadastral offices soon made the clumsy coordinate planimeters obsolete.

Nevertheless, the cone wheel and disk wheel integrators still had their place in calculating mechanisms, and in measurement devices which had to continuously present intermediate data.

3 The Second Generation: No Slipping but Rolling

The early integrating mechanisms had one intrinsic flaw that limited the accuracy of calculation. Precision is restricted due to the fact that the wheel slips over the disk when it is moved in radial direction. This effect also

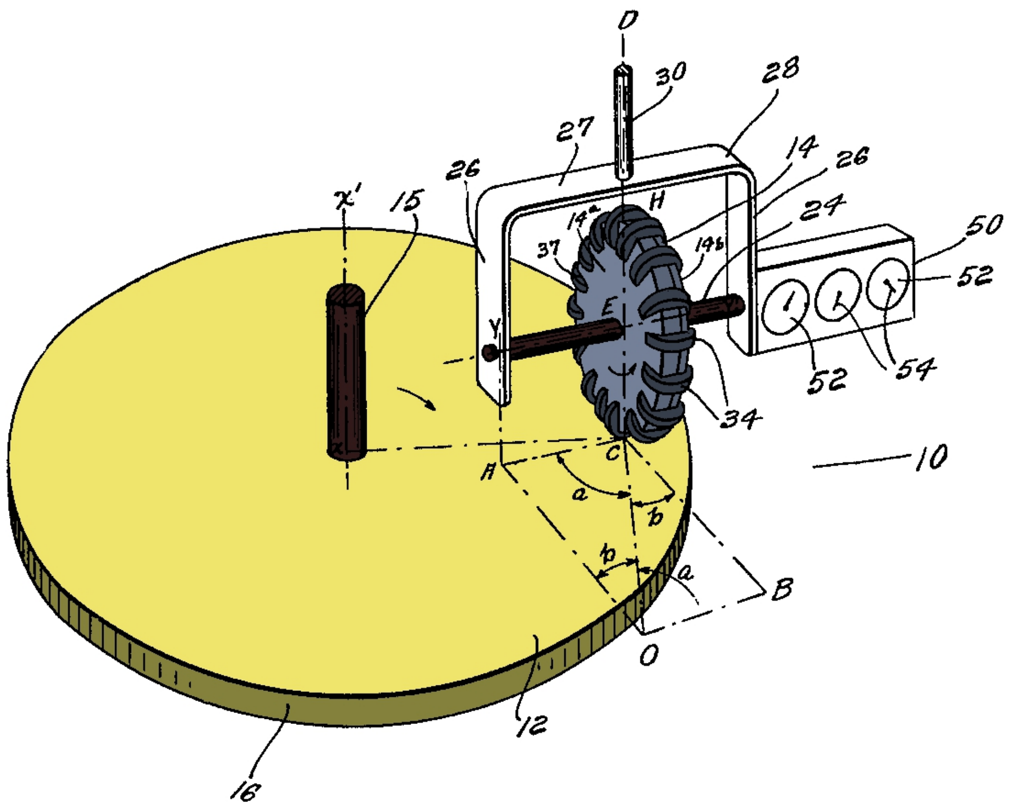


Figure 9: Rollers at the periphery of the wheel to prevent slipping [Con25, modified]

increases abrasion. Furthermore, more power is needed to compensate for the friction loss.

A fine illustration of this problem can be found in a patent that was published about 100 years after Gonnella's first treatise [Con25] (Figure 9).

The inventor pointed out that integrating devices of the disk wheel type "...have heretofore been avoided in flow meters commercially, on account of the effect of the slipping or rubbing motion of the periphery of the integrating wheel the relatively large power required to drive said integrating wheel and the excessive force required to actuate the integrating mechanism". He proudly claimed to "...have provided a device easy and cheap to construct, neat in appearance and wherein the integration is more positively actuated than in former types of integrating flow meters." [Con25]. His suggestion was to simply apply small rollers at the perimeter of the wheel to smoothen the axial motion.

This solution was far from "neat in appearance", but it serves perfectly to illustrate the problem.

The friction issue was a familiar problem and had been investigated as far back as in the 1850s.

3.1 Maxwell's Platometer

James Clerk Maxwell, a Cambridge undergraduate at that time, saw the various planimeters at the Great Exhibition and reported that "the mode in which the cone was made to act the part of a wheel of variable radius" of Sang's Planimeter "greatly excited my admiration." [Max55].

Nevertheless, he did not fail to recognise the possible disadvantages: "Having seen and admired these instruments at the Great Exhibition in 1851, and being convinced that the combination of slipping and rolling was a drawback on the perfection of the instrument" he decided to create a planimeter "by which the motion should be that of perfect rolling in every motion of which the instrument is capable." [Max56]. He ended up with a device which consisted of two equal spheres mounted in a framework and rolling on one another.

In an addendum to his paper, dated 30th April 1856, Maxwell admitted that "since the design of the above instrument was submitted to the Society of Arts, I have met with a description of an instrument combining simplicity of construction with the power of adaption to designs of any size, and at the same time more portable than any other instrument of the kind." [Max56]. Maybe the mentioned instrument, which was the polar planimeter invented by Jakob Amsler, was the reason why Maxwell finally refrained from having his invention built.

3.2 Thomson's Integrator

James Thomson, the elder brother of the famous Lord Kelvin, knew the classical cone wheel and disk wheel mechanisms as well as Maxwell's device. He clearly saw their disadvantages: "The kinematic principle for integrating ydx, which is used in the instruments well known as Morin's Dynamometer and Sang's Planimeter, admirable as it is in many respects, involves one element of imperfection which cannot but prevent our contemplating it with full satisfaction. This imperfection consists in the sliding action with its rolling action which the edge wheel or roller is required to take in conjunction with its rolling action, which alone is desirable for exact communication of motion from the disk or cone to the edge roller." The modern polar planimeters did not meet Thomson's demands either: "the very ingenious, simple and practically useful instrument known as Amsler's Polar Planimeter, although different in its main features of principle and mode of action from the instruments just referred to, ranks along with them in involving the like imperfection of requiring to have a sidewise sliding action…"[Tho75a].

Thomson now found a simple and elegant solution for the abolition of slipping. He simply replaced the wheel of a disk wheel integrator with a ball [Tho75a] (Figure 10).

The next problem he then had to face was how to tap the covered distance and feed it into a counter.

Since the ball had to roll freely, it was impossible to attach a counting device to it, but Thomson found another solution. He simply added a cylinder to the disk ball mechanism to which the ball's rotation was

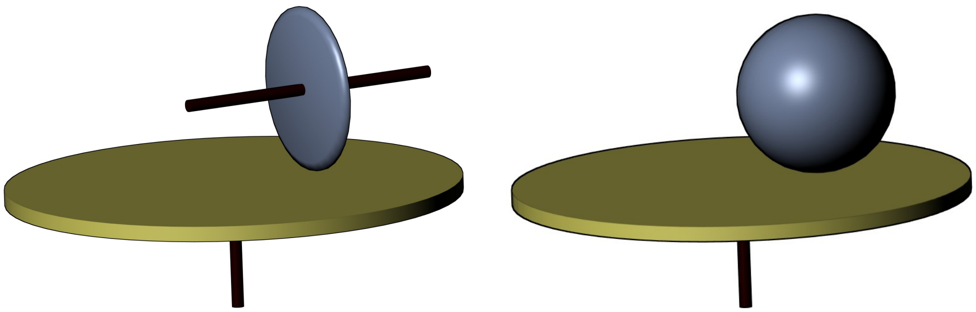


Figure 10: From wheel to ball

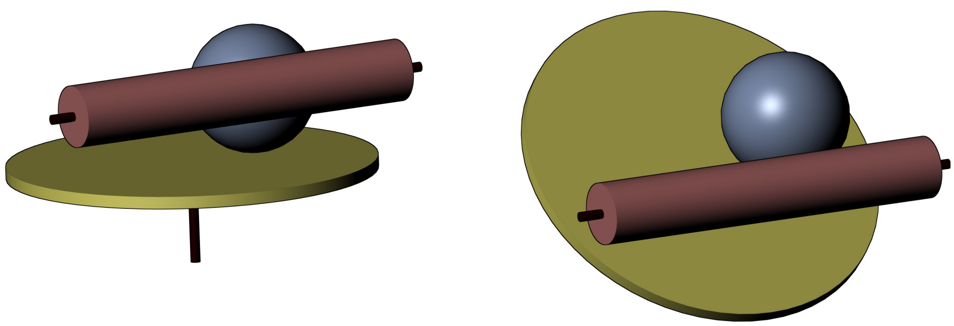


Figure 11: Adding a cylinder

passed. At the cylinder's axis a counter could easily be mounted.

The whole ensemble was arranged in a 45 degree angle, so that the ball was pressed by gravity at both disk and cylinder (Figure 11).

James Thomson's brother William, the later Lord Kelvin, immediately realized the potential of this invention.

In the same volume of the Proceedings of the Royal Society of London in which James Thomson's article "On an integrating machine having a new kinematic principle" was published, a series of articles of Lord Kelvin, covering the theory and describing applications for the instrument, also appeared [Tho75c, Tho75b, Tho75d].

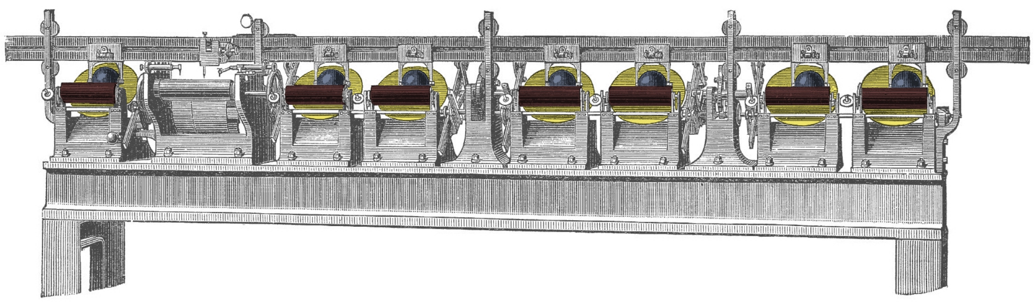


Figure 12: Machine designed by Lord Kelvin and used by the Meteorological Office [SC86, modified]

Kelvin not only suggested combining two integrating mechanisms by feeding the output of one disk ball mechanism into another, he also went one step further with an idea that came to him as a "pleasing surprise": "Compel agreement between the functions fed into the double machine and that given out by it. This is to be done by establishing a connexion which shall cause the motion of the centre of the globe of the first integrator of the double machine to be the same as that of the surface of the second integrator's cylinder. The motion of each will necessarily be a solution of (1). [...] it seems to me very remarkable that the general differential equation of the second order with variable coefficients may be rigorously, continuously, and in a single process solved by a machine" [Tho75c, p. 270]. What Kelvin had invented was a feedback loop for solving differential equations.

In another treatise he also suggested to connect any number of these devices and combining them by linkages, therefore being able to integrate any differential equation of any order and offering "...a complete mechanical integration of the problem of finding the free motions of any number of mutually influencing particles, not restricted by any of the approximate suppositions which the analytical treatment of the lunar and planetary theories requires" [Tho75b].

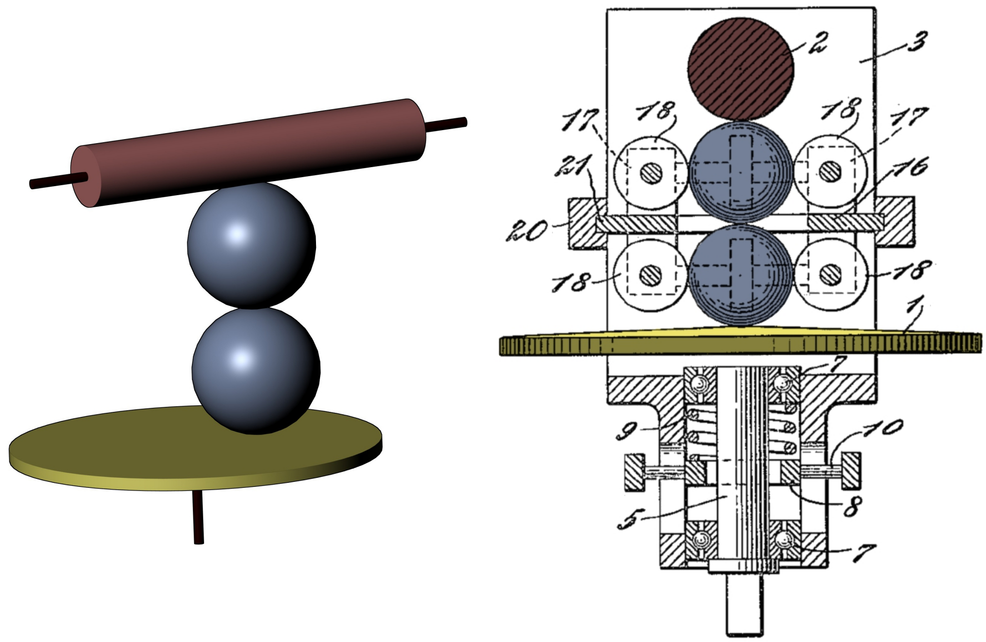
The first prototype consisting of a number of disk ball integrators was constructed and handed over to the Meteorological Office. After several mechanical modifications it was used there as a harmonic analyzer for thermometric and barometric data (Figure 12). The prototype was composed of seven disk ball integrators to compute the Fourier coefficients up to third order in the expression $f(x) = \frac{\alpha_0}{2} + \Sigma_{k\geq 1}(\alpha_k \sin(kx) + \beta_k \cos(kx))$ by evaluating the integrals $\alpha_k = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(kx)\, \mathrm{d}x$

and $\beta_k = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(kx)\,\mathrm{d}x$ [SC86, Tho81, Otn07]. Later machines were built "...to substitute brass for brain in the great mechanical labour of calculating the elementary constituents of the whole tidal rise and fall" [Tho81], that is, to analyse measured water level data for tide prediction.

3.3 Ford's Mechanism

Nevertheless, the Kelvin integrators were not perfect. Especially when connecting several integrators, the input of the last integrator required more torque than the output the previous one could deliver. This output torque was limited by the friction between the ball and the disk, which is proportional to the weight of the ball. Therefore, something had to be invented to increase the force pressing the ball against the disk beyond the level of gravity.

A big step forward in the development of integrating mechanisms can be seen in an US-Patent granted to Hannibal Ford [For19]. Ford improved the integrating mechanism by simply placing a second ball on top of the first. The two balls were held in position by a cage, pressed together and towards the recording cylinder by tight springs. Now the torque at the output was



(a) Principle of Ford's mechanism (b) Ford's mechanism [For19]

Figure 13: Adding a second ball

proportional to the force of the springs, so the mechanism was less prone to precision errors induced by small disturbance forces (Figure 13). This double ball integrator even produced enough torque at the output to drive other integrators.

This opened opportunities for many applications.

4 Practical Applications

While integrating mechanisms underwent a rise in status from measuring tools to calculating instruments, devices that could continuously totalize the product of two variables found also profane utilisation in upcoming industries. Their applications were diverse.

4.1 Range Keepers

In 1915, Hannibal Ford, an engineer and former employee of the Sperry Gyroscope Company, founded the Ford Marine Appliance, which, in the same year, became the Ford Instrument Company [Cly02], a company that produced fire control mechanisms for the US Navy. In 1916, Ford's first range keeper was delivered to the Navy. A *Range Keeper* is an analog fire control system used on war vessels that predicts the position of a moving target and corrects the direction of the gun, thereby taking into account the time the projectile takes until it hits its target. Its heart was a very specialized but far from simple analog computer. Ford's range keeper included the double ball integrating mechanism as integrating device together with a lot of gears and levers for other types of calculation (Figure 14).

The range keeper device in fire control computers had

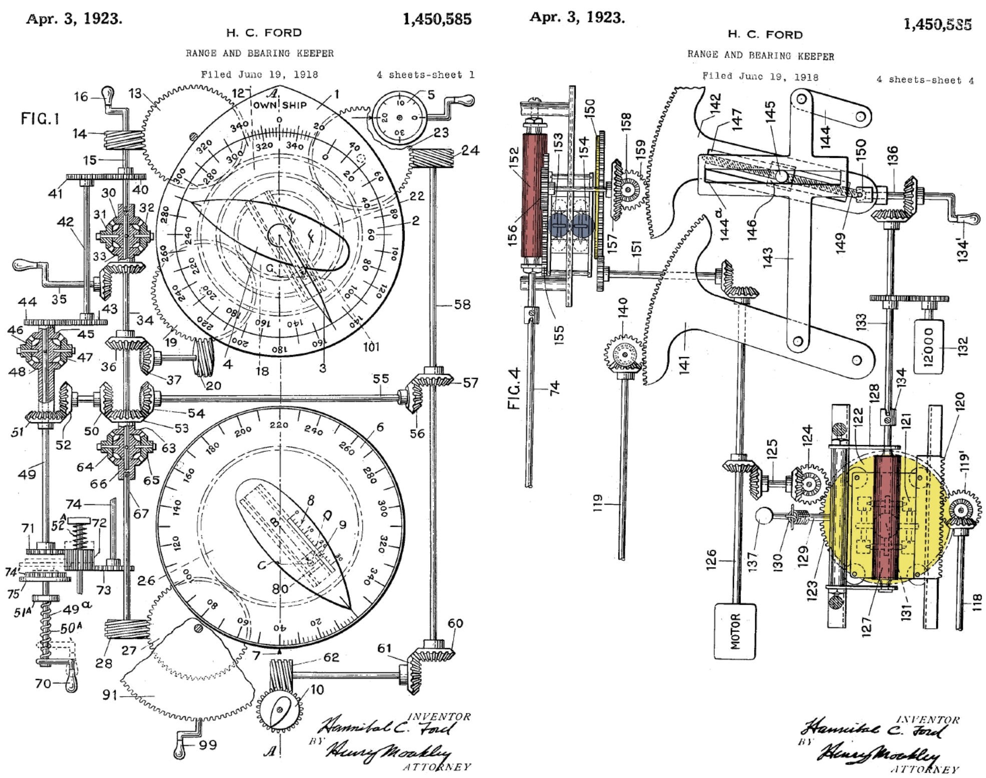
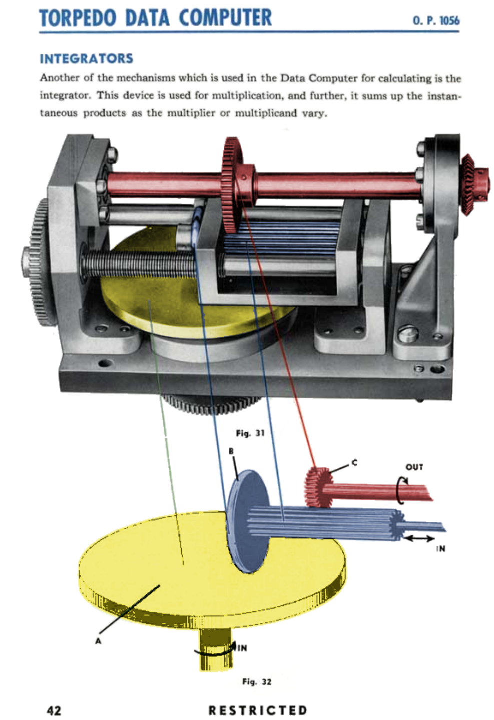


Figure 14: Ford's range keeper [For23, modified]

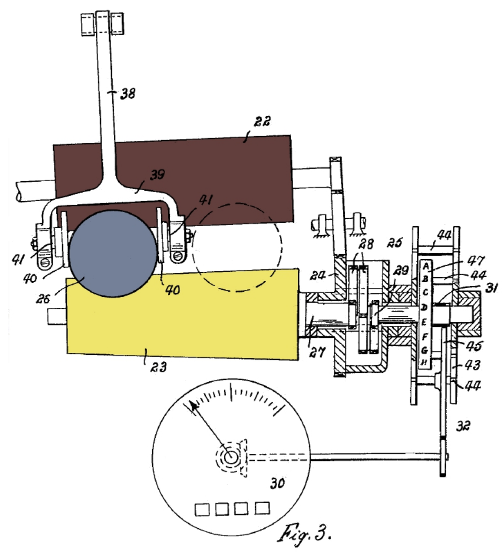
to give a correct result of an integration for each point of time during the calculation, a demand that could not be fulfilled by the widely used polar planimeter, which doesn't provide the result until the end of the operation.

4.2 Train Travelling Times

When setting up train timetables it is neccessary to know not only the travelling time for the whole distance,



(a) Disk wheel integrator in a range keeper of the Arma Corporation [Arm44]



(b) Cone ball integrator in a flow meter [Bro49]

Figure 15: Applications for integrating mechanisms

but for all intermediate stations where the train has to stop. The profile of the railway line determines the forces effecting the train's acceleration for each position along the track. The integral of the acceleration over time results in the speed. The integral of the speed results in the position of the train. And the position along the railway line yields the gradient of the track, making the feedback loop complete. So with just two integrators it is possible to determine a time schedule of a train for a specific railway line.

4.3 Flow Meters

Wherever it is necessary to measure the amount of goods passing continuously by with variable speed, this measuring task needs some sort of integrator. Another main application for the disk wheel mechanism and its relatives is the field of flow meters for different purposes as, for example, in weighing conveyor belts or in meters for measuring an amount of gas, water or any other liquid (Figure 15b).

5 Scientific Analog Supercomputers

In 1842, Professor Moseley had already suggested using his dynamometer for calculations instead of measurements. His intention was to produce numerical tables of functions that could be evaluated by integration, a task to be performed with his analog integrator. Moseley emphasized that "there is a large class of functions whose analytical integration lies beyond the existing resources of mathematical science, the mechanical integration of which this machine would nevertheless effect" [Mos42a]. AS a result, Moseley abstracted the mechanism from technical needs, and positionsit in a pure mathematical context.

About 30 years later Lord Kelvin even extended this idea from simple integrations to systems of differential equations. He first solved these equations iteratively, using a two stage integrator chain of his brother's disk ball cylinder integrators. He manually fed its output again as input into the machine until after several iteration steps a solution was reached with sufficient accuracy.

"But then came a pleasing surprise". Kelvin suggested mechanizing this iteration process by connecting the output of the second integrator with the input of the first [Tho75c]. This feedback loop established a kind of short circuit performing all the iteration steps at once, i. e. solving the differential equation directly.

Kelvin had already perceived the possibility to use a system of integrating mechanisms for solving differential equations of any order [Tho75b]. However, the number of integrators in that integrator chain was limited by friction losses.

Solving these problems and putting Kelvin's ideas into practice is closely related to the work of Vannevar Bush and Douglas Hartree.

5.1 Vannevar Bush

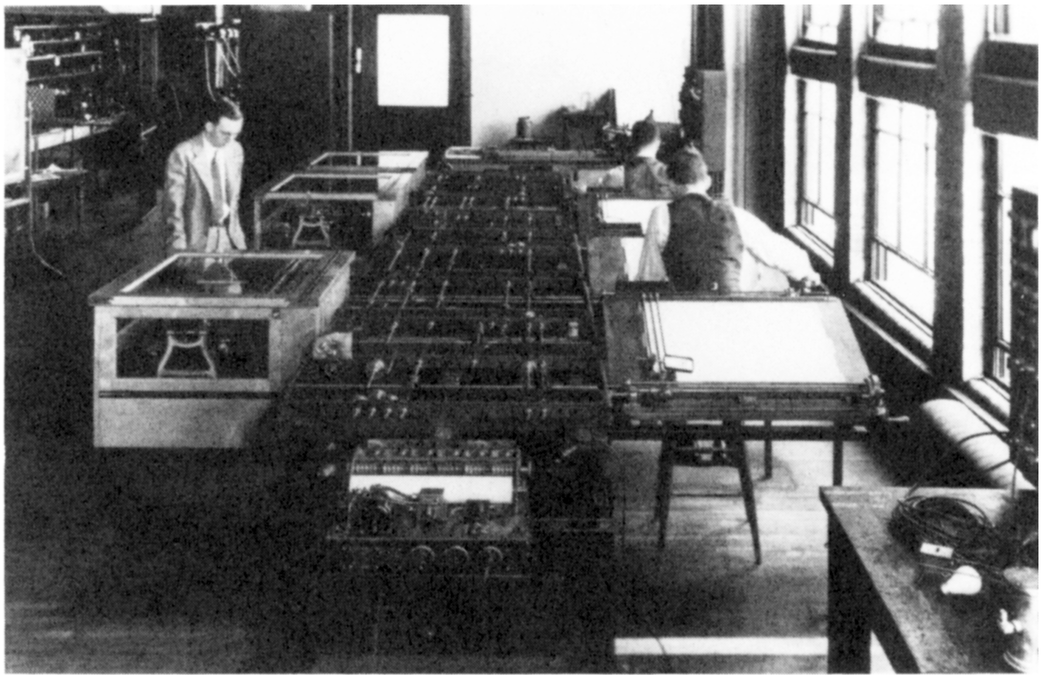


Figure 16: The 1930 Differential Analyzer [Owe86].

In his master's thesis Vannevar Bush had already invented a profile tracer that made use of an integrating gear to graph topographical profile by pushing a gearbox on bicycle wheels along the path of interest. The drawing of the patent specification shows that in 1912 Bush had aready used a wheel disk integrator to integrate the slope of the terrain to calculate its level [Bus12].

In their first equation solving machine, the product integraph, Bush and his team used a modified watthour meter as integrator [BGS27].

This device was soon enhanced by a second disc wheel type integrator. The possibility of changing connections between the integrators by gears with different transmission ratios, or by differential gears to sum up two values, made this instrument applicable to a plenitude of mathematical and physical differential equations, limited only by the order of those equations and their degree of freedom i. e. limited by the number of the integrators in the instrument [BH27]. As a result, plans were soon made to build a new machine with six disk wheel integrators [Bus31].

When using wheels instead of double spheres, the problem of the weak output torques of the integrators reappeared. Bush decided to use torque amplifiers driven by an electric motor.

Following a suggestion of Professor Waldo V. Lyon, Bush coined the term *Differential Analyzer* for this device [Bus31, Owe86], however “this machine neither differentiates nor analyses“ [Har38].

5.2 Douglas Hartree

One of the first to follow the ideas of Bush was Douglas Hartree at the University of Manchester. In his own words he described how he was taken by the idea of a differential analyzer:

"The first photographs I saw of Dr. Bush's machine gave me the feeling that someone had been enjoying himself with a super-Meccano set, and this gave me the idea of trying to build, so far as possible from standard Meccano parts, a model to illustrate the principles of the machine. This originally done more for amusement than with any serious purpose, but the first results were successful beyond my expectations, and suggested that it would be practicable to build such a model to do serious work on problems for wich high accuracy was not required in the results. Largely with the help of Dr. Porter, such a model was built, and proved successful for such work (...). It has also been useful for demonstration purposes, for trying in an experimental form new ideas such as the special input table."[Har40].

In 1935, a full scale differential analyzer was in operation at the University of Manchester with four integrators [Har35] and was soon expanded to eight integrators [Har38].

5.3 Other Successors

The need for solutions of differential equations, both in theoretical science and in practical gunnery asked for more differential analyzers. Over the next few years many smaller and bigger analyzers were built, mainly in western Europe and in the USA. Many of them profited from the experiences at the MIT as Bush was very cooperative and provided drawings [Har35] or even lent a consultant [Owe86]. Table 1 gives a not complete overview of some of the differential analyzers at scientific institutes, at military labors and – due to the Meccano toy – even at a grammar school.

Lord Kelvin had already suggested using a differential analyzer to solve the three body problem [Tho75b]. In astronomy these three bodies are typically sun, moon and earth revolving around each other due to gravity. The principle is the same as for the train schedule: The gravity force is proportional to the acceleration, which is integrated over time to yield the speed, and another integration over the speed results in the position of the heavenly bodies. These positions are again the input to the law of gravity which completes the feedback loop of the differential equations. In contrast to the problem of train schedules not only one train but three bodies are involved, and their accelerations and speeds do not have just one value but three components each. This means that for the solution of a complete three body problem in three dimensions $3 \times 3 \times 2 = 18$ integrators

Intg.	Year	Institution	References
1 (2)	1925	MIT	[Sor54] [BGS27] [BH27]
6	1930	MIT	[Sor54] [Bus31] [MzC49] [Har35] [Ros39] [Wil51]
3 (4)	1934/35	University of Manchester	[Sor54] [Har38]
10	1935	University of Pennsylvania	[Sor54] [Har35]
10	1935	Aberdeen Proving Center, Maryland	[Sor54]
4 (8)	1935	University of Manchester	[Sor54] [Har38] [HN38] [MzC49] [Har35] [Ros39]
5	1936	University of Cambridge	[Sor54] [Har38]
4	1937	Queen's University, Belfast	[Sor54] [Har38] [MzC49] [Wil51]
	1937	Geräteentwicklung Danzig GmbH	[MzC49]
	$\leq$ 1938	Macclesfield Grammar School	[Har38]
8	1939	University of Cambridge	[Sor54] [Har38] [Wil51]
6		Leningrad	[Sor54] [MzC49] [Ros39] [Wil51]
6	1940	Institute of Actuaries	[Sor54]
18 (30)	1942	MIT	[Sor54] [MzC49]
2	1942	Askania, Berlin	[Sor54] [MzC49]
12	1944	Astrophysical Institute, Oslo	[Sor54] [Har38] [MzC49] [Ros39] [Wil51]
3	1949	Yale University	[Sor54] [MB49]
2×6	1952	University of California	[Sor54] [Sor52]
14		General Electric Company	[Sor54]
14		University of California	[Sor54]
		University of Illinois	[Sor54]

Table 1: Differential Analyzers

are necessary. It is therefore no surprise that one of the largest differential analyzers was built with 18 integrators [Owe86, p. 16]. Financed by the Natural Science

Division of the Rockefeller Foundation [Owe86, p. 17] this huge machine was installed again at the MIT. It was planned to extend its capacity to 30 integrators, but as the abilities of digital computers soon exploded, there were cheaper options to solve differential equations.

Finally, the 18 integrator machine from 1940 marks the culmination as well as the end of the evolution of a simple kinematic principle. The unavoidable destiny of all these differential analyzers was the same as for the very first integrating planimeter of Hermann: It is now scrap metal, dismantled and, if lucky, on show in a museum.

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